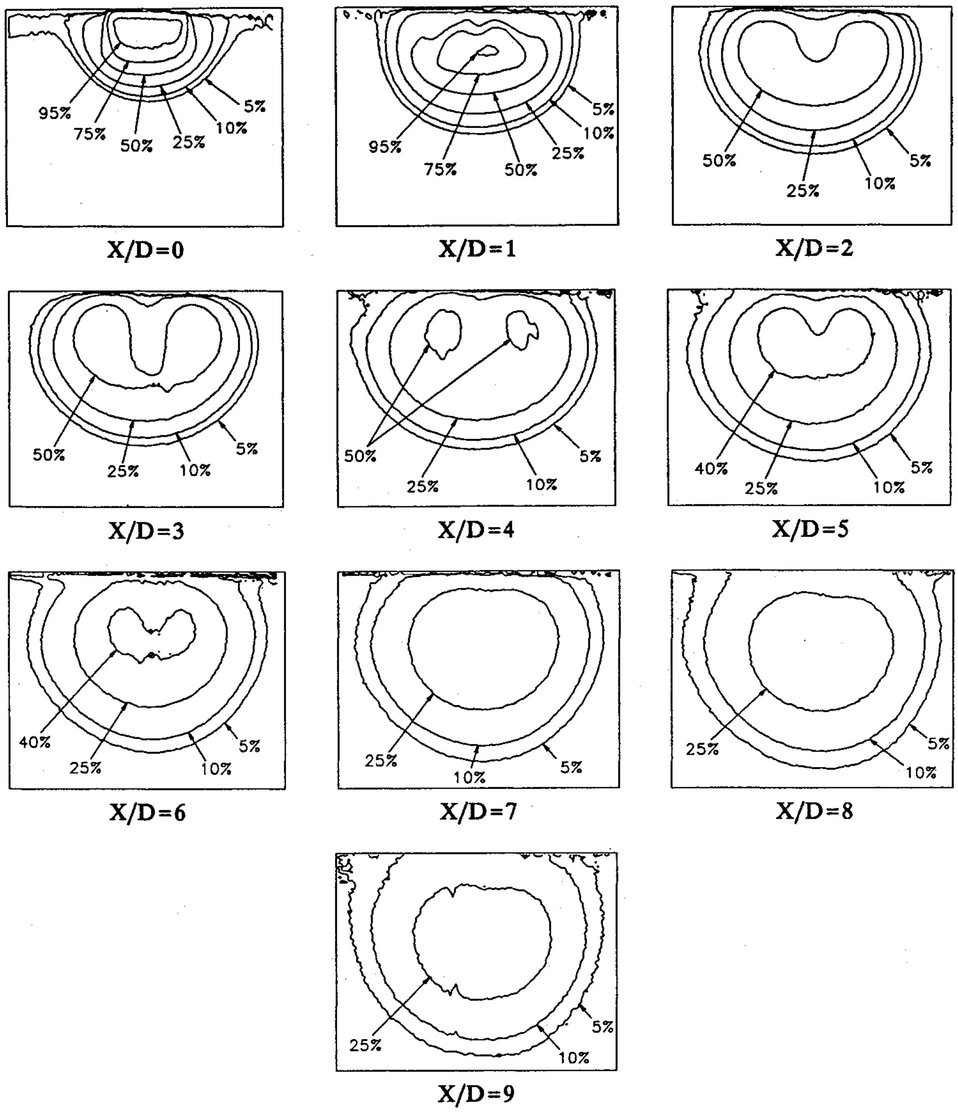

Fig. 4 60-deg injector crossflow injectant mole fraction distribution.

include efforts to quantify the tradeoff between total pressure loss and increases in mixing rate.

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Theoretical Upper Limits on Enthalpy Rocket Performance

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Nomenclature

A, B = intermediate variables for integration
A_t = nozzle throat area, m^2

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C_p = constant pressure specific heat, J/kg K
F = thrust, N
$\dot{H}$ = rate of enthalpy change, J/s
I_{sp} = specific impulse, s
M = molecular (molar) mass of the expellent, kg/mole
m = mass, kg
$\dot{m}$ = mass flow, kg/s
P_0 = stagnation pressure, Pa
R_u = universal gas constant, 8.3143 J/mole K
T_0 = flow stagnation temperature, K
t, t_0, t_b = time, s
γ = ratio of specific heat at constant pressure to specific heat at constant volume
ΔQ_f = heat of fusion, J/kg
ΔV = change in velocity, m/s

Introduction

INTEREST in high-efficiency, reusable rocket engines has kindled interest in the concept of an enthalpy engine. Simply stated, the enthalpy engine uses noncombustive heat transfer from a preheated mass (thermal capacitor) to a pressurized expellent. The concept allows independent selection of expellent and capacitor materials. This holds the promise of specific impulses comparable to those of nuclear rockets without the dangers of dealing with nuclear material. Since the capacitor is not expended, such an engine would also be reusable.

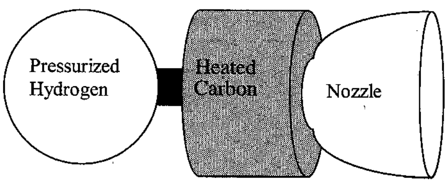

Fig. 1 Simple configuration of pressurized hydrogen passed through a heated carbon mass, then expanded through a nozzle.

Unlike its close cousin, the nuclear rocket, an enthalpy engine's heat source is limited by the heat capacity of its capacitor. The pairing of expellent and capacitor substances is investigated here with the purpose of determining the maximum possible ΔV from each pair. There appears to be an analytical solution to this investigation, which yields dismal results.

Enthalpy Rocket Concept

Simply stated, an enthalpy rocket is a heated mass (thermal capacitor) that transfers its stored thermal energy into a nonreactive expellent, much like a nuclear rocket. Unlike a nuclear rocket, an enthalpy engine employs no nuclear fuel to maintain its core heat. Once the expellent flow begins, the core gradually cools down, and engine efficiency degrades. The true artistry in investigating this concept lies in material selection for both expellent and capacitor.

A simple inspection of the mass flow equation encourages designers to search for maximum operating temperatures in capacitors and minimize molecular masses in expellents:

$$\dot{m} = (P_0 A_t/\sqrt{T_0})\sqrt{(M\gamma/R_u)[(\gamma + 1)/2]^{(\gamma+1)/(1-\gamma)}} \quad (1)$$

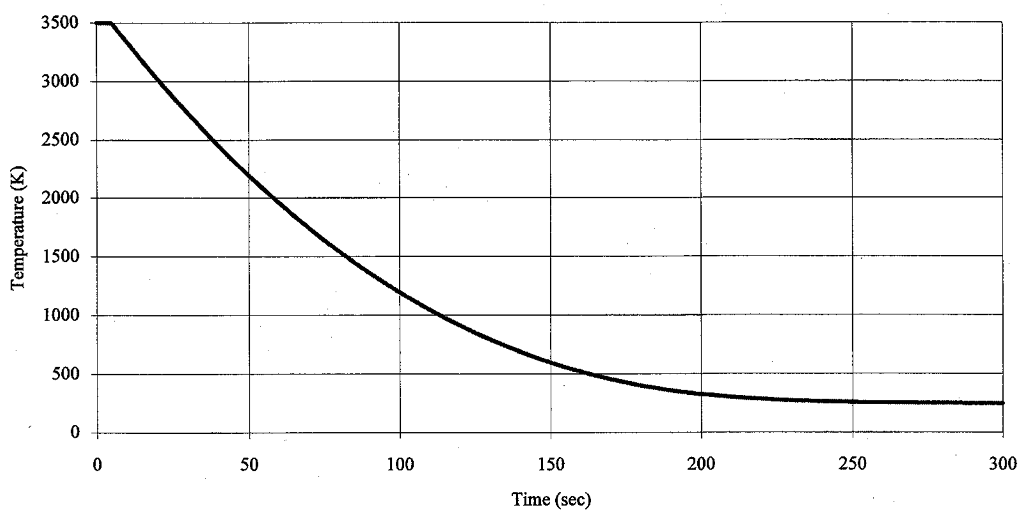

Fig. 2 Temperature decrease of a 100-kg carbon core exposed to a 0.0361-kg/s hydrogen flow.

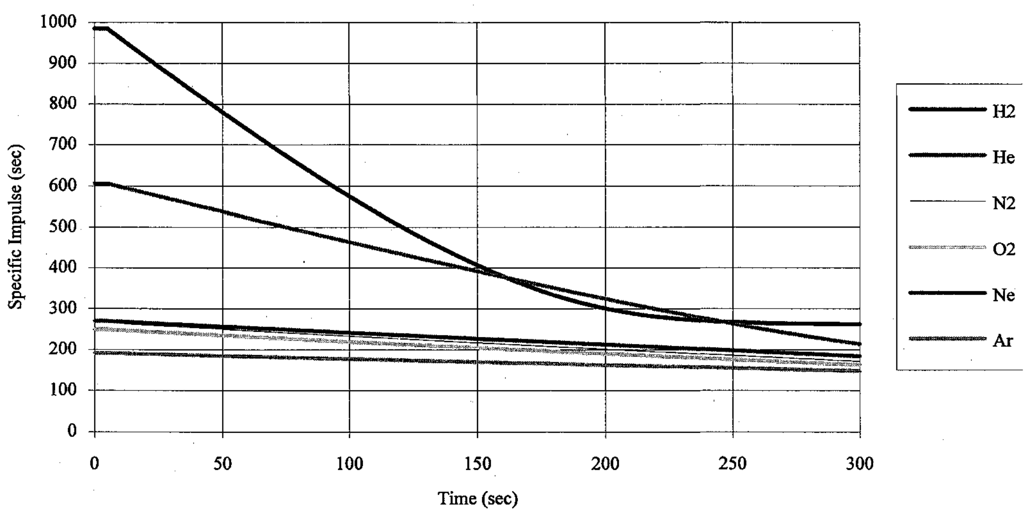

Fig. 3 Specific impulse decrease of other expellent flows through a 100-kg carbon core.

Table 1 Thermodynamic properties[4] of other elements and compounds[a]

Capacitor material	Specific heat, J/kg K	Heat of fusion, J/kg	Melting point, K
Lithium	3582	664.12×10^3	476.95
Beryllium	1825	1089.40×10^3	1576.15
Boron	1026	2053.10×10^3	2377.15
Carbon	709	——	3848.15
Sodium	1228	114.81×10^3	395.95
Magnesium	1023	372.49×10^3	948.15
Aluminum	897	395.96×10^3	958.15
Silicon	705	1412.00×10^3	1708.15
Titanium	523	437.44×10^3	1966.15
Interesting compounds			
Al_2O_3	——	1072.60×10^3	2318.15
BeO	——	2847.96×10^3	2832.15
CaO	——	913.84×10^3	2980.15
MgO	——	1923.22×10^3	2915.15
BeSi[11]	——	1812.00×10^3	1363.15

[a]Carbon's heat of fusion is omitted since nothing remains that could contain it once melted.

Pure hydrogen is an obvious first choice for an expellent. The material comprising the thermal capacitor determines the upper limit on operating temperature. To maintain a solid structure capable of encasing a high-pressure flow, the capacitor must have a very high melting temperature. Running through the periodic table by melting points yields carbon as the first choice, with its melting point of 3848.15 K.

An illustrative example of an enthalpy engine might therefore be pure hydrogen expelled through a carbon core (Fig. 1) heated to near melting (e.g., 3500 K). With no nuclear material, this simple engine offers an exceptional specific impulse. Assuming perfect heat transfer, a modest operating pressure of 2 MPa and infinite nozzle expansion readily yields an I_{sp} of 1016.9 s. Herein is the lure of enthalpy engines. With just a heated mass of carbon, truly outstanding I_{sp} appear possible. (With actual nozzle dimensions, pressures, and nonideal heat transfer, I_{sp} still range between 700–800 s.)

Effects of a Finite Capacitor

This exceptional efficiency is valid only when enthalpy engines first begin operation. Thereafter, as the thermal capacitor cools, the engine's temperature and ultimately its specific impulse both decrease. Modeling the enthalpy increase of the expellent is a simple matter of integrating pure hydrogen's specific heat. Passing this through a 100-kg mass of carbon at a rate of 0.0361 kg/s (chosen for a large expansion ratio nozzle) over a 300-s firing shows a precipitous temperature drop. Ignoring temperature gradients within the finite mass (perfect heat conduction), the carbon temperature drops to the inlet temperature (set at 278 K) in less than 4 min. As the capacitor cools, the total temperature of the expellent drops accordingly (Fig. 2). Over the same duration, a hydrogen flow's I_{sp} dwindles to less than 300 s, scarcely that of a cold gas thruster.

It is evident that hydrogen's chief drawback is its huge C_p, more than twice that of helium or other heavier gases. Heavier gases make lower demands on the capacitor, but overall display poor specific impulses (Fig. 3).

Evaluating capacitors requires a different approach than that used for expellents. An ideal capacitor substance has a very high melting temperature and a high specific heat. Again, the periodic table and CRC[4] yield thermodynamic properties (Table 1).

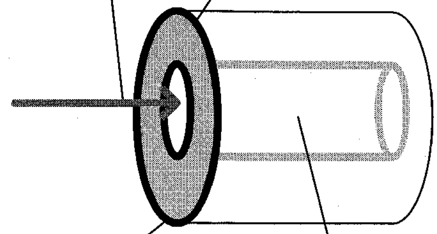

Fig. 4 Basic layout of a nonhomogeneous capacitor. The casing is most likely graphite or tungsten (each melts at over 3700 K) with a melted capacitor within.

The specific heat of the capacitor material is important, but in the carbon example it was assumed to be a homogeneous block. Note the middle columns of Table 1. The thermal energy liberated when a liquid changes phase to a solid can be three to four orders of magnitude greater than that of heat from a cooling solid. The most likely choice for a capacitor is therefore one made of two materials: 1) a high melting point solid capable of containing a liquid capacitor and high-pressure expellent gas and 2) a high melting point material with a high heat of fusion (Fig. 4).

Such a configuration offers a high heat capacity and a constant operating temperature during the phase change.

Total ΔV

If the expellent and capacitor materials are chosen, then the thrust duration determines the required masses of each. Assuming a completely ideal engine that has just enough capacitor mass to heat an expellent flow for the planned burn time, and just enough expellent to last for the burn, the total ΔV possible from any pair is an integrable expression. Assume that an engine can be constructed of massless tankage and ducting. This leaves only the masses of the expellent and capacitor. Begin with simple physics and integrate the acceleration over the duration of the burn time t_b to find the change in velocity. Since the only mass is that of the capacitor m_c and the expellent m_e, assume the thrust is constant (using a phase-change capacitor):

$$\Delta V = F \int_{t_0}^{t_b} \frac{1}{m_c + m_e} \, dt \tag{2}$$

The minimum capacitor mass is just enough to supply adequate enthalpy for the burn duration:

$$m_c = \dot{H} t_b / \Delta Q_f \tag{3}$$

where $\dot{H}$ is the rate of enthalpy added to the expellent flow, and ΔQ_f is the heat of fusion of the solidifying capacitor. The remaining expellent is simply the mass flow (a constant be cause of the constant operating temperature) multiplied by the remaining burn time:

$$m_e = \left[\frac{P_0 A_t}{\sqrt{T_0}} \sqrt{\frac{M\gamma}{R_u} \left(\frac{\gamma + 1}{2} \right)^{(\gamma+1)/(1-\gamma)}} \right] (t_b - t) \tag{4}$$

Combining these two mass terms and rewriting:

$$\Delta V = F \int_{t_0}^{t_b} \frac{1}{\left\{ \frac{\dot{H}}{\Delta Q_f} + \left[\frac{P_0 A_t}{\sqrt{T_0}} \sqrt{\frac{M\gamma}{R_u} \left(\frac{\gamma + 1}{2} \right)^{(\gamma+1)/(1-\gamma)}} \right] \right\} t_b - \left[\frac{P_0 A_t}{\sqrt{T_0}} \sqrt{\frac{M\gamma}{R_u} \left(\frac{\gamma + 1}{2} \right)^{(\gamma+1)/(1-\gamma)}} \right] t} \, dt \tag{5}$$

Table 2 Maximum ΔV for likely expellents through each capacitor

Capacitor material	Maximum ΔV, km/s H_2	He	N_2	O_2	Ne	Ar
Lithium	0.57226	0.71445	1.19603	1.20173	1.05817	1.10197
Beryllium	0.30048	0.44514	0.85873	0.86587	0.83146	0.98701
Boron	0.41622	0.64084	1.16340	1.17085	1.16642	1.36014
Sodium	0.14792	0.18625	0.43950	0.45582	0.59963	0.43206
Magnesium	0.15641	0.22471	0.48425	0.49146	0.44431	0.55087
Aluminum	0.16461	0.23646	0.50608	0.51329	0.46509	0.57418
Silicon	0.36504	0.54204	1.01138	1.01781	0.98696	1.15116
Titanium	0.10380	0.16090	0.34257	0.34938	0.33779	0.44187
Interesting compounds						
Al_2O_3	0.22403	0.35004	0.69158	0.70109	0.69240	0.85878
BeO	0.50737	0.79394	1.39165	1.39648	1.41273	1.62315
CaO	0.16009	0.26014	0.52360	0.53160	0.54347	0.68799
MgO	0.33889	0.54030	1.00732	1.01576	1.02634	1.23337
BeSi	0.54720	0.77770	1.34158	1.33975	1.28857	1.41925

which is of the form:

$$\Delta V = F \int_{t_0}^{t_b} \frac{1}{At_b - Bt}\, dt \tag{6}$$

Completing the integration, inserting the limits, and a bit of algebra yields

$$\Delta V = \frac{F[\ell n(A) - \ell n(A - B)]}{B} \tag{7}$$

Note that the thrust duration t_b is no longer in the function at all! Substituting in the values for A and B gives

$$\Delta V = F\left(\frac{\ell n\left\{\frac{\dot{H}}{\Delta Q_f} + \left[\frac{P_0 A_t}{\sqrt{T_0}}\sqrt{\frac{M\gamma}{R_u}\left(\frac{\gamma+1}{2}\right)^{(\gamma+1)/(1-\gamma)}}\right]\right\} - \ell n\left(\frac{\dot{H}}{\Delta Q_f}\right)}{\frac{P_0 A_t}{\sqrt{T_0}}\sqrt{\frac{M\gamma}{R_u}\left(\frac{\gamma+1}{2}\right)^{(\gamma+1)/(1-\gamma)}}}\right) \tag{8}$$

In actuality, even the choice of P_0 and A_t has a negligible effect on total ΔV. The chosen capacitor substance sets ΔQ_f and T_0. The expellent choice determines the other values. This relation is simple enough that ΔV can be readily tabulated for prospective pairs (Table 2).

These values for maximum ΔV are enough to kill any enthusiasm sparked by high specific impulses. The only combinations that approach a practical orbital transfer vehicle are those using heavier gases. Once calculated out, their capacitor masses are small compared to the required expellent mass. These types of engines have very low I_{sp}, only minimally using the heat from their capacitors to warm a large amount of expellent. The assumptions and idealizations made in these calculations are very optimistic, with perfect heat conduction, and 100% functional engine mass (no payload). Incorporating models of nonideal conditions only reduces the total possible ΔV.

Summation

Though an intriguing alternative to nuclear rockets, enthalpy rockets are hampered by the simple thermochemical limitations of matter. Substances not addressed in this study are welcomed by the authors. Anything that melts above 2000 K and possesses a heat of fusion of over 2000 kJ/kg is a promising candidate for study. However, Eq. (8) is an unavoidable cap to the performance of any design. Though not completely unworthy of study, enthalpy rockets employing any materials addressed here do not appear to hold much promise in applications requiring sizable ΔV or large thrust impulses.

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